647510 **BSc5MatC501x** **Seat No :________**

B.Sc. Semester - 5 (Remedial) (*CBCS*) Examination
March/April-2024 (NEW COURSE)
Mathematical Analysis -1 & Abstract Algebra -1(Core)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que-1(A) **Answer any two out of three.** (10)

(1) Define discrete metric and show that it is a metric on any nonempty set.

(2) X is a metric space. Show that intersection of finite number of open subsets of X is open. Explain with example that this statement is not true for infinite number of open subsets of X.

(3) Explain Cantor set and define it. Show that $\frac{1}{10} \in C$.

Que-1(B) **Answer any one out of two.** (04)

(1) (i) In usual notation show that $(A \cup B)' = A' \cup B'$.

(ii) Explain with example that $\overline{A \cap B} \neq \overline{A} \cap \overline{B}$

(2) (i) Consider $\mathbb{R}$ with discrete metric. Find N(2,1) in $\mathbb{R}$.

(ii) Check whether set $E=(-\infty,2) \cup \{10\}$ is open and/or closed.

Que-2(A) **Answer any two out of three.** (10)

(1) Prove that any bounded function f defined on [a,b] is R-integrable iff For $\forall\ \varepsilon>0$ there exist partition P of [a,b] such that $U(P,f) - L(P,f) < \varepsilon$, Where $\|P\| < \delta$.

(2) Show that product of two bounded R-integrable functions is R-integrable.

(3) Function f is bounded in [a,b]. P and P^* be two partitions of [a,b] Such that $P \subset P^*$, then show that $L(P,f) \leq L(P^*,f)$.

Que-2(B) **Answer any one out of two.** (04)

(1) Find $\int_1^3 (2-3x)\,dx$ using definition of R-integration.

(2) If bounded function f on [a,b] is monotonic then show that f is R-integrable in [a,b].

Que-3(A) **Answer any two out of three.** (10)

(1) Using Darboux theorem find value of

$$\lim_{n\to\infty} \frac{1}{n}\left[\left(\frac{1}{n}\right)^2 + \left(\frac{2}{n}\right)^2 + \dots\dots\dots\dots + \left(\frac{n}{n}\right)^2\right]$$

(2) State and prove fundamental theorem of integral calculus.

(3) Define a group and show that it satisfies both cancellation laws.

Que-3(B) **Answer any one out of two.** (04)

(1) Show that $\frac{\pi^2}{6} \leq \int_0^{\pi} \frac{x}{2+\cos(x)}\,dx \leq \frac{\pi^2}{2}$

(2) Show that $G=\left\{\begin{bmatrix}\cos(\alpha) & -\sin(\alpha)\\ \sin(\alpha) & \cos(\alpha)\end{bmatrix} \middle|\ \alpha \in \mathbb{R}\right\}$ forms a group under matrix multiplication.

Que-4(A) **Answer any two out of three.** (10)

(1) State and prove Lagrange's theorem.

(2) Define: Symmetric group. Find number of elements of symmetric group S_n. Also write all elements of S_3.

(3) G is finite group. For $\forall$ a,b $\in$ G show that $O(b^{-1}ab)=O(a)$.

Que-4(B) **Answer any one out of two.** (04)

(1) Let $G=\{(a,b)/a,b\in \mathbb{R}$ and $a\neq 0\}$ and binary operation * defined on G as: $(a,b)*(c,d) = (ac, bc+d)$, $\forall (a,b), (c,d) \in G$. Show that if $H=\{(1,b)/b\in \mathbb{R}\}$ then $H\leq G$ where $(G,*)$ is group.

(2) $\sigma \epsilon$ S_6 where $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 3 & 5 & 4 & 2 & 1 \end{pmatrix}$ then find σ^{-1} and $o(\sigma)$. Also Check whether σ is even or odd.

Que-5(A) **Answer any two out of three.** (10)

(1) Show that isomorphism of groups is a transitive relation.

(2) Prove that every group of prime order is cyclic.

(3) Prove that infinite cyclic groups have exactly two generators.

Que-5(B) **Answer any one out of two.** (04)

(1) For $n\geq 2$, in usual notation show that A_n is normal subgroup of S_n.

(2) Prepare group table for quotient group $Z_6/<3>$.

****